2nd LANCS Workshop on Discrete and Nonlinear Optimisation

School of Mathematics
University of Southampton

Programme:

Wednesday 24th March

13:15-15:15: Session I

I. Aliev:	Feasibility of integer knapsacks: a geometric approach
W. Achtziger:	Global optimization of truss topology with discrete bar areas
S. Nemeth:	Rapid heuristic projection on simplicial cones

15:15-15:45: Coffee break

15:45-17:45: Session II

C. D'Ambrosio:	A feasibility pump for non-convex MINLPs
L. Galli:	Zero-one quadratically constrained quadratic programmes: relaxation versus reformulation
V. Strusevich:	The symmetric quadratic knapsack problem: relaxation, approximation and scheduling applications

Thursday 25th March

09:15-11:15: Session I

K. Kaparis:	Using eigenvectors to generate cutting planes for the max-cut problem
M. Giandomenico:	Cutting planes for the stable set problem by semidefinite programming
J. Gondzio:	Interior point methods can warmstart

11:15-12:15: Poster session (+ coffee)

P. Gonzalez:	A stochastic programming approach to minimization of power consumption in wireless telecommunication
C. Lopez:	A formulation space search for the circle packing problem
J. Luo:	Scheduling and storage management in container terminals
F. Malliappi:	On Ship Scheduling
M. Mesgarpour:	Efficient algorithms for optimizing runway scheduling
M. Smyrnakis:	Sequentially updated probability collectives

12:15-13:30: Lunch

Thursday 25th March (cont.)

13:30-15:30: Session II

M. Kočvara: Solving the sensor network localization problem by PENNON
A. Letchford: Some convex sets related to non-convex quadratic optimisation problems
H. Qi: The nearest low-rank correlation matrix problem and max-cut

15:30-16:00: Coffee break

16:00-17:20: Session III

J. Rückmann: Mathematical programs with complementarity constraints: critical point theory
C. Smith: A mixed-integer nonlinear programming algorithm for insuring critical paths

19:00: Evening meal at the Karali South Indian Restaurant, 180 Burgess Road, Southampton.

Friday 26th March

09:15-11:15: Session I

R. Werner: Generating joint distributions via mathematical programming
H. Xu: Stochastic Nash equilibrium models
G. Mitra: Optimisation models for decision making under uncertainty: their application to financial analytics

11:15-11:45: Coffee break

11:45-12:30: Open discussion [chaired by J. Fliege].

List of Participants:

1. Prof. Wolfgang Achtziger, Faculty of Mathematics, TU Dortmund.
   `Wolfgang.Achtziger (at) tu-dortmund.de`
2. Dr Iskander Aliev, School of Mathematics, Cardiff University.
   `AlievI (at) cardiff.ac.uk`
3. Dr Claudia D'Ambrosio, DEIS, Bologna University.
   `c.dambrosio (at) unibo.it`
4. Fei Fang, CORMSIS, Southampton University.
   `ff1e08 (at) soton.ac.uk`
5. Prof. Jörg Fliege, CORMSIS, Southampton University.
   `J.Fliege (at) soton.ac.uk`
6. Kimonas Fountoulakis, School of Mathematics, Edinburgh University.
   `K.Fountoulakis (at) sms.ed.ac.uk`
7. Dr Laura Galli, DEIS, Bologna University.
   `l.galli (at) unibo.it`
8. Dr Monia Giandomenico, Dept. of Computer Science, L'Aquila University.
   `giandomenico (at) di.univaq.it`
9. Prof. Jacek Gondzio, School of Mathematics, Edinburgh University.
   `J.Gondzio (at) ed.ac.uk`
10. Pablo González, School of Mathematics, Edinburgh University.
    `P.Gonzalez (at) sms.ed.ac.uk`
11. Dr Konstantinos Kaparis, CORMSIS, Southampton University.
    `K.Kaparis (at) soton.ac.uk`
12. Prof. Michal Kočvara, School of Mathematics, Birmingham University.
    `kocvara (at) maths.bham.ac.uk`
13. Prof. Adam N. Letchford, Dept. of Management Science, Lancaster University.
    `a.n.letchford (at) lancaster.ac.uk`
14. Claudia López Soto, Department of Mathematical Sciences, Brunel University.
    `mapgccl (at) brunel.ac.uk`
15. Jiabin Luo, CORMSIS, Southampton University.
    `j.luo (at) soton.ac.uk`
16. Fotini Malliappi, CORMSIS, Southampton University.
    `F.Malliappi (at) soton.ac.uk`
17. Jakub Mareček, School of Computer Science, Nottingham University.
    `jxm (at) cs.nott.ac.uk`
18. Mohammad Mesgarpour, CORMSIS, Southampton University.
    `m.mesgarpour (at) soton.ac.uk`
19. Prof. Gautam Mitra, Dept. of Mathematical Sciences, Brunel University.
    `gautam.mitra (at) brunel.ac.uk`

20. Dr Sándor Németh, School of Mathematics, Birmingham University.
`nemeths (at) for.mat.bham.ac.uk`

21. Dr Andrew Parkes, School of Computer Science, Nottingham University.
`ajp (at) cs.nott.ac.uk`

22. Dr Houduo Qi, CORMSIS, Southampton University.
`H.Qi (at) soton.ac.uk`

23. Eggert Rose, School of Mathematics, Birmingham University.
`rosee (at) for.mat.bham.ac.uk`

24. Dr Jan-J. Rückmann, School of Mathematics, Birmingham University.
`j.ruckmann (at) bham.ac.uk`

25. Prof. J. Cole Smith, Dept. of Industrial and Systems Engineering, Florida University.
`cole (at) ise.ufl.edu`

26. Michalis Smyrnakis, Dept. of Mathematics, Bristol University.
`m.smyrnakis (at) bris.ac.uk`

27. Prof. Vitaly Strusevich, Dept. of Mathematical Sciences, Greenwich University.
`V.Strusevich (at) gre.ac.uk`

28. Dr Ralf Werner, Deutsche Pfandbriefbank AG, Munich.
`werner_ralf (at) gmx.net`

29. Dr Huifu Xu, CORMSIS, University of Southampton.
`H.Xu (at) soton.ac.uk`

Abstracts

Global optimization of truss topology with discrete bar areas

Wolfgang Achtziger
Wolfgang.Achtziger (at) tu-dortmund.de
Faculty of Mathematics, Technical University of Dortmund

We consider the classical problem of finding a truss design with minimal compliance subject to a given external force and a volume bound. The design variables describe the cross-section areas of the bars. While this problem is well-studied for continuous bar areas, we treat here the case of discrete areas. This problem is of major practical relevance bacause trusses usually are manufactured from pre-produced bars with given areas. As a special case, we consider the design problem for a single bar area, i.e., a 0/1-problem.

In contrast to heuristic methods considered in other approaches, we present an algorithmic framework for the calculation of a global optimizer of the underlying large-scaled mixed integer design problem. We present a convergent branch-and-bound algorithm which is based on nonconvex continuous relaxations. The approach benefits from the fact that the relaxed nonlinear optimization problem can be equivalently formulated as a quadratic program (QP). Here the paper extends known theory. Although the Hessian of this QP is indefinite, it is possible to circumvent non-convexity and to calculate global optimizers. Moreover, the QPs to be treated in the branch-and-bound search tree differ from each other just in the objective function. Together with a special purpose heuristic which quickly finds good feasible discrete solutions, this makes the resulting branch-and-bound method very efficient. The talk closes with large-scale numerical examples.

The talk is based on joint work with Mathias Stolpe (Technical University of Denmark).

Feasibility of integer knapsacks: a geometric approach

Iskander Aliev
AlievI (at) cardiff.ac.uk
School of Mathematics, Cardiff University

Given an integer mxn matrix A satisfying certain regularity assumptions, we consider the set F(A) of all integer vectors b such that the associated knapsack polytope P(A,b)=x: Ax=b, x¿=0 contains an integer point. A description of the set F(A) in terms of polynomials that can be regarded as a discrete analog of the celebrated Farkas Lemma was obtained by Lasserre. The test Gomory and Chvatal functions for F(A) were also given by Blair and Jeroslow.

In this talk we discuss the geometric structure of the set F(A) which, apart from a few special cases, remains unexplored. Results of Knight, Simpson and Tijdeman and Pleasants, Ray and Simpson suggest that the set F(A) may be decomposed into the set of all integer points in the interior of a certain translated feasible cone and a complementary set with complex combinatorial structure. For instance, when m=1 the set F(A) is known to contain all consecutive integers greater than the Frobenius number associated with A. We give an optimal, up to a constant multiplier, estimate for the position of a feasible cone in the set F(A) and also prove that a much stronger asymptotic estimate holds on average.

Abstracts

A feasibility pump for non-convex MINLPs

Claudia D'Ambrosio
c.dambrosio (at) unibo.it
DEIS, Bologna University

We present a new Feasibility Pump (FP) algorithm for finding feasible solutions for non-convex Mixed Integer Nonlinear Programming (MINLP) problems, based on iterative solution of two simpler subproblems, i.e., relaxations of the original problem. Difficulties arising from nonconvexities in the models have to be overcome. In this case the first subproblem, obtained by relaxing the integer requirements on the variables, is a nonconvex Nonlinear Program.

For the second subproblem, we propose different approaches: a Mixed Integer Linear Programming (MILP) relaxation; a Mixed Integer Quadratic Programming relaxation; or, following the original idea of FP for MILP, we round the fractional values of the solution of the first subproblem. Computational experiments shows good performance of the algorithms on selected MINLPLib instances.

Joint work with: Antonio Frangioni, Leo Liberti, Andrea Lodi.

Zero-one quadratically constrained quadratic programmes: relaxation versus reformulation

Laura Galli
l.galli (at) unibo.it
DEIS, Bologna University

It has been known for some time that semidefinite programming (SDP) can be used to derive useful relaxations for a variety of optimisation problems, and especially problems with quadratic aspects. Moreover, in the particular case of zero-one quadratic programs, SDP has been used to reformulate problems, rather than merely relax them. The purpose of reformulation is to strengthen the continuous relaxation of the problem, while leaving the optimal solution unchanged. In this paper, we explore the possibility of extending the reformulation approach to the (much) more general case of zero-one quadratically constrained quadratic programs.

This talk is based on joint work with Adam Letchford.

Abstracts

Cutting planes for the stable set problem by semidefinite programming

Monia Giandomenico
giandomenico (at) di.univaq.it
Department of Computer Science, L'Aquila University

A Semidefinite Programming (SDP) relaxation, namely the Lovász theta relaxation, yields strong upper bounds for the stable set problem. The same upper bounds can be obtained by an infinite family of linear inequalities, called orthonormal representation inequalities(ORIs). However, SDP can still be used to both separate and strengthen the ORIs. The resulting cutting plane algorithms are tested on standard benchmark instances and the results are presented.

The talk is based on joint work with Adam Letchford, Fabrizio Rossi and Stefano Smriglio.

Interior point methods can warmstart

Jacek Gondzio
J.Gondzio (at) ed.ac.uk
School of Mathematics, Edinburgh University

Many optimization approaches such as for example, decomposition techniques, Branch and Bound, cutting planes, sequential quadratic programming, etc. require the solution of a sequence of closely related (hence similar) linear or quadratic programming subproblems. In my talk I will discuss the reoptimization techniques for an interior point method applied in this context. Differences between an active set (simplex-type) approach and an interior point approach will be clarified and advantages of both techniques will be discussed.

Using eigenvectors to generate cutting planes for the max-cut problem

Konstantinos Kaparis
k.kaparis (at) soton.ac.uk
CORMSIS, Southampton University

In their landmark paper, Goemans and Williamson showed that semidefinite programming (SDP) yields provably good upper and lower bounds for the max-cut problem. In particular, they show that the ratio between the two bounds is no larger than 1.139.

We are interested in obtaining bounds of similar strength, but without using SDP. The key concept is that the feasible region of the SDP relaxation is defined by an infinite number of linear inequalities, whose separation problem can be solved efficiently via an eigenvalue computation. The inequalities can be used directly as cutting planes within a standard LP-based cutting-plane algorithm, or they can be used as the source of additional inequalities.

The talk is based on joint work with Daniel Grainger and Adam Letchford.

Abstracts

Solving the sensor network localization problem by PENNON

Michal Kočvara
kocvara (at) maths.bham.ac.uk
School of Mathematics, Birmingham University

We propose a hybrid algorithm for the sensor network localization problem. In the first stage, we solve a nonlinear SDP relaxation of the problem to get a close approximation of the global solution. In the second stage, we use this solution as an initial point for an augmented Lagrangian code solving (locally) the original nonlinear programming formulation of the problem. We will demonstrate that, in most cases, this procedure delivers an exact global solution of the sensor network localization problem. Both subproblems are solved by the code PENNON.

Some convex sets related to non-convex quadratic optimisation problems

Adam N. Letchford
a.n.letchford (at) lancaster.ac.uk
Department of Management Science, Lancaster University

If an optimisation problem is linear, then the convex hull of its feasible solutions is almost always a polyhedron. The study of polyhedra associated with hard discrete linear optimisation problems has been very fruitful, providing the basis for effective bounding procedures and exact optimisation algorithms for several such problems.

When it comes to non-linear problems, however, the convex hull is usually not a polyhedron. To study such convex sets, one must supplement polyhedral theory with elements of convex analysis. This talk will present some recent progress in this area, concerned with problems that have non-convex quadratic objective functions.

The talk is based on joint work with Sam Burer, Iowa University.

Abstracts

Optimisation models for decision making under uncertainty: their application to financial analytics

Gautam Mitra
gautam.mitra (at) brunel.ac.uk
Dept. of Mathematical Sciences, Brunel University

Two models of choice under uncertainty, the mean risk model and second order stochastic dominance, are introduced. (i)The discrete extensions of the mean risk model to include threshold constraints and cardinality restrictions are considered. These models are formulated as quadratic mixed integer programming problems; their solution algorithms are also discussed. (ii) Models which use second order stochastic dominance and tail risk measures are presented and the unifying aspects which connect these two approaches are examined. We outline the computational algorithms for the solution of these classes of problems. The results of empirical studies involving these classes of models are discussed. In particular we consider the scale-up properties of the corresponding computational models.

Rapid heuristic projection on simplicial cones

Sándor Németh
nemeths (at) for.mat.bham.ac.uk
School of Mathematics, Birmingham University

A very fast heuristic iterative method of projection on simplicial cones is presented. It consists in solving two linear systems at each step of the iteration. The extensive experiments indicate that the method furnishes the exact solution in more then 99.7 percent of the cases. The average number of steps is 5,67 (we have not found any examples which required more than 13 steps) and the relative number of steps with respect to the dimension decreases dramatically. Roughly speaking, for high enough dimensions the absolute number of steps is independent of the dimension.

Abstracts

The nearest low-rank correlation matrix problem and max-cut

Houduo Qi
H.Qi (at) soton.ac.uk
CORMSIS, Southampton University

In this talk, we propose a computational scheme for the max-cut problem. The scheme consists of two steps. In the first step, we relate the max-cut problem to the nearest rank-1 correlation matrix problem. We use our latest Newton algorithm to solve this non-convex quadratic semidefinite programming problem. The output is a cut.

The second step returns to the max-cut problem and aims to improve the quality of the cut obtained in the first step. The idea is to consider blocks of nodes. Within each block, we define a cut on this block of nodes in such a way that the overall cut is at least as good as the old one. We consider blocks of sizes ranging from 2 to 10. After considering those blocks, our numerical results show no improvement by considering blocks bigger than 10. Finding a cut on each block of nodes can be easily done by using a majorisation method originated from multidimensional scaling.

We tested our scheme against all the test problems in Rudy collection from Big Mac Library. We are able to solve 34 out of 130 problems to optimality. The absolute gaps for the rest problems are very small. The proposed algorithm is also very fast.

Mathematical programs with complementarity constraints: critical point theory

Jan-J. Rückmann
j.ruckmann (at) bham.ac.uk
School of Mathematics, Birmingham University

We study mathematical programs with complementarity constraints from a topological point of view. We derive a Morse Lemma at nondegenerate C-stationary points and prove two basic theorems from Morse theory (deformation theorem and cell-attachment theorem). Outside the C-stationary point set, continuous deformation of lower level sets can be performed and, as a consequence, the topological data (such as the number of connected components) remain invariant. However, when passing a level containing a C-stationary point, the topology of the lower level set changes via the attachment of a q-dimensional cell where its dimension equals the stationary C-index of the corresponding C-stationary point. The stationary C-index depends on both the restricted Hessian of the Lagrangian and the Lagrange multipliers related to bi-active complementarity constraints. Finally, some relations with other stationarity concepts are discussed.

The talk is based on a joint paper with Hubertus Th. Jongen and Vladimir Shikhman (both from RWTH Aachen University, Germany).

Abstracts

A mixed-integer nonlinear programming algorithm for insuring critical paths

J. Cole Smith

cole (at) ise.ufl.edu

Dept. of Industrial and Systems Engineering, Florida University

We consider a stochastic optimization problem involving protection of vital arcs in a critical path network. We analyze a problem in which task finishing times are uncertain, but can be insured a priori to mitigate potential delays. We trade off costs incurred in insuring arcs with expected penalties associated with late completion times, where lateness penalties are lower semi-continuous nondecreasing functions of completion time. We provide decomposition strategies to solve this problem with respect to either convex or nonconvex penalty functions. In particular, we employ the Reformulation-Linearization Technique to make the problem amenable to solution via Benders decomposition. We also consider a chance-constrained version of this problem, in which the probability of completing a project on time is sufficiently large.

The symmetric quadratic knapsack problem: relaxation, approximation and scheduling applications

Vitaly Strusevich

V.Strusevich (at) gre.ac.uk

Department of Mathematical Sciences, Greenwich University

We consider a quadratic knapsack problem (QKP) to minimize a symmetric non-separable function. Problems of this type can serve as mathematical models of various single-machine scheduling problems, e.g., to minimize total weighted completion time with a machine non-availability constraint, to minimize total weighted earliness-tardiness around a common due date etc.

For these scheduling applications the objective function of the QKP appears convex. We show that the continuous relaxation of the QKP reduces to minimizing a quadratic convex flow on a series-parallel network. Further, the continuous solution can be rounded, which results into a constant-ratio approximation algorithm for the QKP and for the relevant scheduling problems. Moreover, for the QKP a fully polynomial time approximation scheme (FPTAS) can be developed and adapted to design FPTASs for the corresponding scheduling problems.

Abstracts

Generating joint distributions via mathematical programming

Ralf Werner
werner_ralf (at) gmx.net
Deutsche Pfandbriefbank AG, Munich

In this talk we will investigate how given univariate (i.e. one-dimensional) distributions can be coupled to a joint multivariate (i.e. multidimensional) distribution. For this purpose we will both investigate the case of discrete and continuous marginals. Although the underlying theory of copulas is well-understood and researched, numerical considerations have been neglected by and large. We will therefore discuss the pros and cons of working with a fully parametric low-dimensional copula compared to a completely non-parametric model-free approach. It will be motivated how the different concepts lead to low-dimensional continuous non-convex optimization problems on the one hand and to huge (integer?) linear programs with special structure on the other hand. We will then show how the discrete case can be generalized to predefined multivariate marginal distributions and we will briefly explain how such a model is applied in the pricing of counterparty credit risk of OTC derivatives.

Stochastic Nash equilibrium models

Huifu Xu
H.Xu (at) soton.ac.uk
CORMSIS, University of Southampton

In this talk we discuss a Nash equilibrium model where the underlying objective functions involve uncertainty and nonsmoothness. The model covers two stage stochastic equilibrium programs with equilibrium constraints (SEPEC) and two stage stochastic generalized Nash equilibrium problem. The well-known Monte Carlo sampling method is applied to solve the problem. Since the underlying functions are not necessarily convex, we look into both Nash equilibrium and Nash stationary point characterized in terms of Clarke generalized gradients. Under some moderate conditions, it is shown that with probability one, a statistical estimator of a Nash equilibrium or a Nash-C-stationary point obtained from sample average approximate equilibrium problem converges to its true counterpart. Moreover, under some calmness conditions of the Clarke generalized derivatives, it is shown that with probability approaching one exponentially fast with the increase of sample size, the estimators converge to their true counterparts. Finally, the model is applied to a stochastic Nash equilibrium problem in the electricity market.