LANCS Workshop on Modelling and Solving Complex Optimisation Problems

Lecture Theatre 5, Lancaster University Management School
Monday 11th – Wednesday 13th April 2011

Programme:

Monday 11th April

13:15-13:45: Arrival and registration

13:45-14:00: Welcome and general information

14:00-15:30: Session I: Tutorials

B. Büke:	Stochastic Linear and Nonlinear Programming
S. Wallace:	Stochastic Mixed-Integer Linear Programming

15:30-16:00: Coffee break

16:00-17:15: Session II: Invited Talks

C. Cartis:	Optimal Newton-type methods for nonconvex smooth optimization
K. Kaparis:	Gap inequalities for non-convex mixed-integer quadratic programmes

Tuesday 12th April

09:15-10:45: Session I: Tutorials

C. D'Ambrosio:	Convex Mixed-Integer Nonlinear Programming
A. Letchford:	Non-convex Mixed-Integer Nonlinear Programming

10:45-11:15: Coffee break

11:15-13:00: Session II: Invited Talks

A. Grothey:	Multi-level crash-start for IPM applied to stochastic programming problems
J. Hall:	High performance computing and the simplex method
L. Galli:	Convexification of mixed-integer quadratically constrained quadratic programs

13:00-14:15: Lunch break

Tuesday 12th April (cont.)

14:15-15:30: Session III: Invited Talks

D. Kuhn Scenario-free approximations to stochastic programming via decision rules
J. Mareček: Semidefinite programming relaxations in timetabling

15:30-16:00: Coffee break

16:00-17:15: Session IV: Invited Talks

K. McKinnon: Stochastic ship routing with inventory management by column generation
M. Noorizadegan: Solving the stochastic location routing problem by decomposition methods

Wednesday 13th April

09:15-10:30: Session I: Invited Talks

H. Qi: A low-rank SDP approach to outlier detection
P. Richtarik: From sparse principal component analysis to compressed sensing

10:30-11:00: Coffee break

11:00-12:15: Session II: Invited Talks

M. Takač: Random coordinate descent methods: applications and complexity
Y. Zhao: The stochastic facility layout problem

12:15-12:30: Summing up and discussion

List of Participants:

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Abstracts

Towards optimal Newton-type methods for nonconvex smooth optimization

Coralia Cartis
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We show that the steepest-descent and Newton methods for unconstrained non-convex optimization, under standard assumptions, may both require a number of iterations and function evaluations arbitrarily close to the steepest-descent's global worst-case complexity bound. This shows that the latter upper bound is essentially tight for steepest descent and that the Newton method may be as slow as the steepest-descent method in the worst case. Next, the cubic regularization of the Newton method (Griewank (1981), Nesterov & Polyak (2006)) is considered and extended to large-scale problems, while preserving the same order of its improved worst-case complexity (by comparison to that of steepest-descent). This improved worst-case bound is also shown to be essentially tight. We further address the optimality of cubic regularization from a worst-case complexity point of view amongst a class of second-order methods. An extension of cubic regularization to bound-constrained problems will be presented that satisfies the unconstrained function-evaluation complexity bound of cubic regularization.

Joint work with Nick Gould (Rutherford Appleton Laboratory, UK) and Philippe Toint (Namur University, Belgium).

Convexification of mixed-integer quadratically constrained quadratic programs

Laura Galli
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In this study we explore when and how one can obtain convex reformulations of mixed-integer quadratically constrained quadratic programs (MIQCQP). In particular, we show that semidefinite programming (SDP) can be used both to guarantee convexity of MIQCQP functions, and to perturb them in such a way that the lower bound obtained from the continuous relaxation of the problem is equal to the SDP bound (sometimes even better). A serious problem is however posed by continuous variables that have quadratic terms in one or more constraints. Such variables can cause convex reformulations to be weak, or even prevent them from existing at all. This is not surprising, however, given that non-convex QCQP, a purely continuous problem, is an $\mathcal{NP}$-hard global optimisation problem.

Joint work with Adam Letchford (Lancaster University).

Abstracts

Multi-level crash-start for interior-point methods applied to stochastic programming problems

Andreas Grothey
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We propose two different crash-starting techniques for interior point methods, applicable to stochastic programming problems.

In the first case, a series of simplified problems with fewer scenarios (obtained from scenario reduction) is solved, to successively obtain an estimate of an advanced point on the central path of the full problem. In the second case, only the first-stage components of such an advanced central point are obtained, by performing half of an iteration of a decomposition scheme on the barrier problem corresponding to the deterministic equivalent.

We analyse the conditions under which such a scheme is successful, argue that it leads to improved complexity, and give numerical results obtained by the IPM solver OOPS.

High performance computing and the simplex method

Julian Hall
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When solving families of related LP problems and many classes of single LP problems, the simplex method is the preferred computational technique. There is, therefore, considerable motivation for exploring how the simplex method may exploit modern desktop high performance computing (HPC) architectures. This talk will discuss two approaches by which this may be achieved on multi-core CPUs and many-core GPUs. The first is to identify LP problem structure that allows the revised simplex method to exploit such architectures, and the second is to develop novel algorithmic variants of the simplex method to facilitate HPC implementations for general LP problems. Results will be presented for two CPU and one GPU implementation, and the scope for exploiting combined CPU and GPU systems will be discussed.

Abstracts

Gap inequalities for non-convex mixed-integer quadratic programs

Konstantinos Kaparis
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A standard tool for tackling integer programming and combinatorial optimisation problems is to define an associated family of polyhedra and then derive strong valid linear inequalities (cutting planes) for those polyhedra. Since the 1980s, it has been known that the polyhedron associated with unconstrained quadratic programming in 0-1 variables, the so-called boolean quadric polytope, is essentially the same (modulo an affine mapping) to the polyhedron associated with the max-cut problem, the so-called cut polytope.

Laurent & Poljak (1996) defined an extremely general class of valid inequalities, called gap inequalities, for the cut polytope. Using the above mentioned mapping, one can define gap inequalities for the boolean quadric polytope. We show that, in fact, one can define in a natural way gap inequalities for general mixed-integer quadratic programs. These gap inequalities seem to make useful cutting planes for non-convex instances.

Joint work with Laura Galli (Bologna) and Adam N. Letchford (Lancaster).

Scenario-free approximations to stochastic programming via decision rules

Daniel Kuhn
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Linear stochastic programming provides a flexible toolbox for analyzing real-life decision situations, but it can become computationally cumbersome when recourse decisions are involved. The latter are usually modeled as decision rules, i.e., functions of the uncertain problem data. It has recently been argued that stochastic programs can quite generally be made tractable by restricting the space of decision rules to those that exhibit a linear data dependence. In this talk we propose an efficient method to estimate the approximation error introduced by this rather drastic means of complexity reduction: we apply the linear decision rule restriction not only to the primal but also to a dual version of the stochastic program and show that both arising approximate problems are equivalent to tractable conic programs of moderate sizes. Moreover, we propose a lifting technique that maps a given stochastic program to an equivalent problem on a higher-dimensional probability space. We prove that solving the lifted problem in primal and dual linear decision rules provides tighter bounds than those obtained from applying linear decision rules to the original problem.

Abstracts

Semidefinite programming relaxations in timetabling and matrix-free implementations of augmented Lagrangian methods for solving them

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In order to derive strong bounds for many timetabling and scheduling problems, one should consider more than the graph colouring component. We present semidefinite programming relaxations of graph colouring with an upper bound on the number of uses of each colour and extensions encountered in timetabling. The bound on the number of uses of a colour stems naturally from the number of rooms or machines available. The extensions consider room sizes, room features, room assignment stability, and pre-allocated room assignments. Such relaxations can be solved efficiently using alternating direction augmented Lagrangian methods (ALM). Goldfarb and Ma have shown the rate of convergence of ALM is asymptotically the best possible, with respect to the error, among first-order methods. On the example of bounded graph colouring, we present a "matrix-free" implementation of an ALM, exploiting the structure of the matrices across all linear algebraic operations involved, and a subsequent randomised rounding routine. The computational results suggest this may turn out to be the method of choice in practical timetabling.

Solving a stochastic ship routing problem with inventory management using column generation

Ken McKinnon
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This talk will describe a solution approach for a stochastic ship routing problem with inventory management. The problem involves finding a least cost set of trips for a fleet of ships transporting a single commodity when the demand for the commodity is uncertain. Storage at consumption and supply ports is limited and inventory levels are monitored in the model. The demand rate for a period is not known until the beginning of that period and is assumed to be constant within the period. When ship speeds are constant this is a stochastic MILP and when they are variable it is a stochastic MINLP.

The talk will describe a Branch and Price solution approach, in which each pricing subproblem is a stochastic routing problem for a single ship, which is solved by stochastic DP, and each column in the master problem corresponds to a stochastic trip plan for a single ship. Some computational results will be given.

Abstracts

Solving the stochastic location-routing problem by decomposition methods

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The stochastic location-routing problem (SLRP) is one of the most important problems in supply chain management. It is concerned with decisions including location of facilities, flow of product between suppliers, facilities and customers, allocation of suppliers to facilities and facilities to customers and design of routes under demand uncertainty.

Here, we view the SLRP as a two-stage stochastic mixed-integer programme (2-SMIP), in which the first stage (strategic stage) is essentially a facility location problem and the second stage (tactical stage) is a kind of vehicle routing problem.

Among the algorithms developed for solving general 2-SMIP problems, disjunctive programming has been applied successfully. Here, we attempt to take advantage of the special structure of the SLRP: by studying the convex hull of the solutions to the vehicle routing problem in the second stage, one can derive useful optimality cuts for the first stage problem.

Joint work with Bo Chen (Warwick University).

A low-rank SDP approach to outlier detection

Houduo Qi
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CORMSIS, Southampton University

This is a preliminary study on efficient algorithms for detecting outliers in a given data set of size n. The study proposes to use $L1$ regression to find the best subset of size k ($< n$). This naturally leads to a quadratic integer programming problem. We then use the well-known matrix lifting technique to reformulate this problem as a low-rank SDP. To prevent unboundedness of the feasible region, we introduce a constraint that is related to the least-squares regression. We are able to derive a worst-case bound on the deviation of the derived solution from the optimal one. Moreover, we can recover the least-squares regression by simply tightening the introduced constraint. However, the numerical challenge is how to solve the low-rank SDP efficiently. We report our preliminary results on some selected examples in statistics.

Abstracts

From sparse principal component analysis to compressed sensing: computing sparse approximations to extreme eigenvectors

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We study the problem of finding, simultaneously, jointly sparse approximations to the eigenvectors associated with the smallest and largest eigenvalues of a symmetric positive semidefinite matrix. This problem is equivalent to the Compressed Sensing problem of finding bounds on the asymmetric Restricted Isometry Constants with the additional new requirement for the respective sparse extreme eigenvectors to be supported on the same set.

We propose an l_1-penalized optimization formulation of the problem, analyze a simple alternating direction method for solving a convexified version of it, give conditions for when this convexification is exact, and offer geometrical interpretation of the origin of joint sparsity in the algorithm in terms of projections of lines onto hypercubes.

In the non-penalized case, the iterates of our algorithm are identical to the normalized gradients of the iterates of the Cauchy steepest descent method applied to minimizing a convex quadratic function. This sheds light on a crucial step in the convergence analysis of the latter by Akaike [1]. Our algorithm can also be thought of as an extension of the method of Journée *et al.* [2], where the authors are interested in the largest eigenvalue only, which the authors apply to sparse principal component analysis.

[1] H. Akaike, *On a successive transformation of probability distribution and its application to the analysis of the optimum gradient method*, Annals of the Institute of Statistical Mathematics 11:1 (1959) 1–16.

[2] M. Journée, Yu. Nesterov, P. Richtárik and R. Sepulchre, *Generalized power method for sparse principal component analysis*, Journal of Machine Learning Research 11 (2010) 517–553.

Abstracts

Random coordinate descent methods: applications and complexity

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We develop a randomized coordinate descent method for minimizing the sum of a smooth and a simple nonsmooth block-separable convex function (e.g., $\frac{1}{2}\|Ax - b\|_2^2 + \gamma\|x\|_1$) and prove that the method obtains an ϵ-accurate solution, with probability at least $1 - \rho$, in at most $O((n/\epsilon)\log(n/\epsilon\rho))$ iterations, where n is the dimension of the problem. This simplifies, improves and extends recent results of Nesterov [2/2010].

This algorithm is efficient for huge and sparse problem instances. Such instances arise, e.g., in truss topology design, compressed sensing or support vector machines (SVM). For huge dimension even a valuation of objective function can be expensive! Therefore our algorithm take advantage of a special problem structure which makes it efficient. We present also our preliminary result for sparse regression problem with matrix with dimension $10^9 \times 10^8$ and application in SVM.

The stochastic facility layout problem

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Facility layout problems (FLPs) are studied to determine locations for facilities (machines) within a factory to achieve an optimal material handling or transportation costs. When factories face uncertain demands, especially change of demand levels, the nature of the FLP becomes stochastic. In this research, such a stochastic FLP is formulated as a layout-flow model which consists of two components, i.e. a layout problem and a flow assignment problem, to address the only uncertainty in demand levels. A heuristic can be found to transform the stochastic layout-flow model to a deterministic quadratic assignment problem (QAP) so that good solutions to the original stochastic facility location model can be found given that this deterministic QAP can be solved by various existing methodologies. Examples show such transformation gives much efficiency in finding solutions for stochastic FLP and reduces computational effort to a good extent. Future research would include other uncertainties such as change in the types of products for consideration.