THE CAPITAL COSTS AND REWARDS OF INVENTORIES IN THE SUPPLY CHAIN OF INDEPENDENT ACTORS

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Setting

- \( P = R - C = w \gamma - c \gamma - A(X) - K \)
- Assume known and constant:
  - \( w \): selling price / product
  - \( c \): variable costs / product
  - \( \gamma \): product sales volume rate / time
  - \( K \): fixed costs
- \( X \): set of decision variables
- Then:
  - \( \text{Max } P(X) = \text{Min } A(X) \)
- \( A(X) \) ?
Example 1: EOQ

\[ A_1 = AC(Q_1) = \frac{s_1 \gamma}{Q_1} + h_1 \frac{Q_1}{2} \]

- **Fixed set-up cost (\(\text{\£/order}\))**
- **Unit holding cost (\(\text{\£/product, time}\))**

\[ h_1 = \alpha (w_1 + e_1) + f_1 \]

\[ H = \frac{1}{T} \int_0^T h(t)I(t)dt \]
\[ H = \frac{h}{T} \int_0^T I(t)dt = hE(I) \]

- \(\alpha\) Discount rate
- \(w_1\) Purchase price (\(\text{\£/prod.}\))
- \(e_1\) Value added costs (\(\text{\£/prod.}\))
- \(f_1\) Out-of-pocket costs (\(\text{\£/prod.}\))

Unit holding costs are based on costs (investment)
Supply Chain (SC) optimum

\[ \forall i: P_i = w_{i-1} y - (w_i + e_i) y - A_i (X) - K_i \]

\[ \text{Max } P_i (X) = \text{Min } A_i (X) \]

\[ \text{Max } \Sigma_i P_i (X) = \text{Min } \Sigma_i A_i (X) \quad \text{(SC optimum)} \]

\[ A_i (X) \]
Approach 1: Buyer-vendor supply chain

Assume: Vendor produces lot-for-lot at finite production rate $R$, zero lead-times

**Classic I**
(Goyal 1976, Banerjee, 1986, Joglekar 1988, Weng 1995, ... Huang et al. 2011)

\[
AC_1(Q_1) = \frac{s_1 y}{Q_1} + h_1 \frac{Q_1}{2}
\]

\[
h_1 = \alpha_1 (w_1 + e_1) + f_1
\]

\[
AC_2(Q_1) = \frac{s_2 y}{Q_1} + h_2 \left(\frac{y}{R}\right) \frac{Q_1}{2}
\]

\[
h_2 = \alpha_2 (w_2 + e_2) + f_2
\]

Derive SC optimum from
\[
AC = AC_1 + AC_2
\]
Assume: Each echelon produces an integer multiple $m_i$ of $Q_{i-1}$ at infinite production rate $R$

**Approach 2: Multi-echelon theory**

Classic II
(Clark and Scarf 1960, Chen et al. 2001)

Derive SC optimum from either:
(1) Installation stocks or
(2) Echelon stocks

Unanswered: what are the individual $A_i$?
Approach 2: Multi-echelon theory

**Installation stocks**

Total cost function of the SC

\[ AC = \sum_{i=1}^{n} AC_i = \sum_{i=1}^{n} \left[ s_i \frac{y}{Q_i} + h_i E(I_i) \right] \]

Average inventory levels (installation stocks)

\[ E(I_i) = \frac{m_i-1}{2m_i} Q_i \text{ for } i \in N \setminus \{1\}, \text{ and } E(I_1) = \frac{Q_1}{2}. \]

Unit holding costs

\( h_i = \text{based on the investment made by value added activities at echelon } i \text{ plus all the value already added in the upstream echelons } i+1 \text{ to } n \)

Any transfer prices, if charged, are to be ignored for calculating \( h_i \)  
Clark (1958)
**Approach 2: Multi-echelon theory**

**Echelon stocks**

Total cost function of the SC

\[ AC = \sum_{i=1}^{n} \overline{AC}_i = \sum_{i=1}^{n} \left[ s_i \frac{y}{Q_i} + \tilde{h}_i E(\tilde{I}_i) \right] \]

Average inventory levels (installation stocks)

\[ E(\tilde{I}_i) \equiv Q_i \frac{Q_i}{2} \]

Unit echelon holding costs for \( i \)

\( = \) based on the value added by converting product from echelon \( i+1 \) to echelon \( i \)

\[ h_i = \sum_{j=i}^{n} \tilde{h}_j, \forall i \in N, \tilde{h}_i = h_i - h_{i+1}, \forall i \in N \setminus \{n\}, \tilde{h}_n \equiv h_n. \]

Conclusion 1: AC derived from installation stocks = AC derived from echelon stocks

Conclusion 2: \( AC_i \neq \overline{AC}_i \).

**Unanswered:** What is the ‘real’ AC function for each echelon?
Approach 3: Crowther (1967)

- For 2-echelon situations
- Assume that supplier:
  - supplies to many buyers
  - has always sufficient inventory to draw from
  - determines his own replenishment policy based on constant overall demand => this cost is fixed
- Supplier revenue potential from each buyer is:
  \[ AC_2 = \alpha (w_1 - w_2) Q_1/2 = h' Q_1/2 \]
- Not much applied, mis-interpreted as a holding cost (Gurnani 2001, Wang 2005)
Unanswered questions

- Contradictions between Classic I, II, and III:
  - 3 different SC functions?
  - 3 different holding costs for each of the firms?

- Hence:
  - What is the profit function of each of the firms?
  - What is the profit function of their collective (SC)?
  - What is the relationship between these functions?

- Suppose
  - Each echelon is an independent firm
  - Transfer prices are charged (£/product), to be paid at each transaction
A collective of independent firms

- $N$ independent firms $1, \ldots, i, \ldots, n$
- Cash-flow function of firm $i$

\[
a_i(t, X) = b_i(t, X) + \sum_{j \neq i}^n (w_{ji}(t, X) - w_{ij}(t, X)),
\]

where $w_{ji}(t, X)$ represents the cash-flow from $j$ to $i$ ($i, j \in N, i \neq j$) at time $t$, $b_i(t, X)$ is the cash-flow of $i$ with the outside world at $t$, and $X$ is the set of decision variables available to $N$, influencing both the size and the timing of the cash-flows.
Axioms

Axiom 1. For any \( i, j \in N \) and \( X \), any payments of volume \( w(t, X) \) made by \( i \) to \( j \) at time \( t \) results in a volume \( w(t, X) \) that arrives at \( j \) on time \( t \).

Axiom 2. The Net Present Value NPV\(_f\)(\( X, \alpha_f \)) of a firm \( f \) is \( \int_0^\infty a_f(t, X)e^{-\alpha_f t}dt \), where \( a_f(t, X) \) is the cash-flow function of the firm, and \( \alpha_f \) the continuous rate of return (at the opportunity cost of capital).

Axiom 3. If a collective of firms wish to determine the strategy \( X^* \) that maximises the NPV of the collective as given by Axiom 2, they have to agree on the value of \( \alpha \).

Even if these axioms would not hold, a thought experiment could be conducted in which they are assumed to hold. What is then the consequence?
Theorem 1

Theorem 1. A collective of n firms adopting Axioms 1-3 and NPV as the measure to take decisions, derives its optimal policy $X^*$ from the function $\text{NPV}_N(X, \alpha)$ given by:

$$\text{NPV}_N(X, \alpha) = \int_0^\infty \sum_{i=1}^n b_i(t, X) e^{-\alpha t} dt = \sum_{i=1}^n \text{NPV}_i(X, \alpha), \quad (2)$$

Proof. Due to Axiom 1, the cash-flow function $a_N(t)$ of the collective is given by $a_N(t) = \sum_{i=1}^n a_i(t) = \sum_{i=1}^n b_i(t, X)$, and due to Axiom 2, $\text{NPV}_N(X, \alpha) = \int_0^\infty \sum_{i=1}^n b_i(t, X) e^{-\alpha t} dt$. If the firms adopt Axiom 3, then $\alpha_i = \alpha, \; i \in N$, and applying the sum rule of integration gives:

$$\sum_{i=1}^n \text{NPV}_i(X, \alpha) = \sum_{i=1}^n \int_0^\infty \left[ b_i(t, X) + \sum_{j \neq i}^n (w_{ji}(t, X) - w_{ij}(t, X)) \right] e^{-\alpha t} dt$$

Thought experiment

Classic I: in violation
Classic II: correct SC function, but incomplete (function of each firm?)
Classic III: in agreement (but limited in applicability)
Towards a `solution’

• Adopt Axioms 1-3 as a working framework
• Apply NPV, i.e. the profit function of each firm is the Laplace transform of its cash-flows (Grubbstrom 1967)
• Use Annuity Stream (AS) profit functions \( (\text{AS}(X) = \alpha \text{NPV}(X)) \)
• Use linear approximations of the AS functions to identify any `missing’ or `incorrect’ terms in the Classic approaches
• What is already known:
  • Average cost functions are linear approximations of the AS function (Hadley and Whitin 1963)
  • Capital cost of inventories can be derived from AS functions (Grubbstrom 1980)
What is not entirely understood

- There can be different NPV solutions for the same system
- To allow a fair judgement of the classic inventory models from an NPV-perspective, this flexibility in NPV should be recognised
- Importance of the ANCHOR POINT (AP) in the system

**Definition 1. Anchor Point.** The Anchor Point in an (NPV) decision model is an arbitrary moment in the future, chosen to coincide with the start or end of some activity, that does not change with a change in any of the policy variables or other parameters in the model.

(Deullens and Janssens, 2011)

- **Example**: BATCH sales EOQ model
Example: Batch sales EOQ – NPV (1)

NPV solution of Grubbström (1980)

\[
AS_0 = (wyT e^{-\alpha T} - s) \left[ \sum_{i=0}^{\infty} \alpha e^{-i\alpha T} \right] - c(y)y
\]

\[
\overline{AS}_0 = (w - c(y))y - \frac{sy}{Q} - \alpha w \frac{Q}{2} - \alpha \frac{s}{2}
\]

\[
Q_0^* = \sqrt{\frac{2sy}{\alpha w}}
\]

Grubbström (implicitly) placed the AP at start of production = PUSH conditions

The classic function is valid IF the unit holding cost is valued at sales price
Example: Batch sales EOQ – NPV (2)

Placing the AP at start of sales (= PULL conditions) produces an alternative NPV solution (Beullens and Janssens, 2011)

\[ Q^*_L = \sqrt{\frac{2sy}{\alpha(2c(y) - w)}} \]

otherwise \( Q^*_L \to \infty \)

Unit holding costs are still to be valued at cost price, but the supplier’s reward is missing from the classic solution.

\[
AS_L = (wyTe^{-\alpha L} - se^{-\alpha(L-T)})\left(\sum_{i=0}^{\infty} \frac{\alpha e^{-i\alpha T}}{\alpha^2}ight) - c(y)y e^{-\alpha(L-T)}
\]

\[
\overline{AS}_L = \left[(w - c(y))y - \frac{sy}{Q} - \alpha c(y)\frac{Q}{2} + \alpha(w - c(y))\frac{Q}{2} - \alpha \frac{s}{2}e^{-\alpha L}\right]
\]

The AP method shows that multiple objective functions and hence optimal solutions are possible from an NPV model.
Serial multi-echelon supply chain

- Set of (arbitrary) (transfer) prices
  \[ W = \{ w_i \mid w_i \in \mathbb{R}, i \in N \cup \{0\} \} \]

- Set of (arbitrary) lead-times, constant or function of lot-sizes
  \[ \Lambda = \{ L_i \mid L_i \in \mathbb{R}^+, i \in N \cup \{0\} \setminus \{n\} \} \]

- Set-up costs \( s_i \) for production, material handling, transport
- Variable costs \( e_i \) for value added production, material handling, transport
- Out-of-pocket costs for stock keeping (insurance, renting of warehouse space, ...)

Let \( I_i(t) \) be the stock at echelon \( i \in N \) at any time \( t \) (excluding any stock in transit). Define \( \bar{I}_i(t) \equiv \sum_{j=1}^{i} I_j(t) \) as the echelon stock for \( i \in N \cup \{0\} \). (An alternative is to define the echelon stock including stock in transit, but this is less useful here.) Let \( h_i(w_j) \equiv \alpha_i w_j \), for \( i \in N \), and \( j \in N \cup \{0\} \), where \( \alpha_i \) is the discount rate adopted by echelon \( i \). Define \( h_i = h_i(w_i) + \alpha_i e_i + f_i \) as the unit holding costs for echelon \( i \in N \), and \( \hat{h}_i \equiv h_i - h_i(w_i) = \alpha_i e_i + f_i \) as the unit echelon holding costs. Define the echelon lead-time for \( i \in N \) as \( \bar{L}_i \equiv \sum_{j=0}^{i-1} L_j \). (Following conventions, \( \sum_{j=x}^{y} Z_j \equiv 0 \) whenever \( x > y \), \( \forall Z \in \mathbb{R} \); thus \( \bar{L}_0 \equiv 0 \), and \( \bar{I}_0 \equiv 0 \).)
The serial multi-echelon SC

Figure 2  Cash-flows proof of Theorem 2 (AP at start of production)
Results

**Theorem 2.** If each echelon \( i \in N \) adopts Axioms 1-2 and NPV as the measure to take decisions, the linear approximation of its annuity stream profit function, for any \( Q, \Lambda, \) and \( W, \) and when the AP is set at the start of production for echelon \( k \in N, \) is given by:

\[
AS^k_i \approx \left( (w_{i-1} - w_i - e_i)y - s_i(1 + \alpha_i(\bar{L}_i - \bar{L}_k)) \frac{y}{Q_i} - \alpha_i \frac{s_i}{2} - [h_i + \alpha_i f_i(\bar{L}_i - \bar{L}_k)]E(\bar{I}_i) \
+ h_i(w_{i-1})E(\bar{I}_{i-1}) - (h_i - f_i)y(\bar{L}_i - \bar{L}_k) + h_i(w_{i-1})y(\bar{L}_{i-1} - \bar{L}_k) \right)e^{-\alpha_i L}
\]

where \( E(\bar{I}_0) = 0, \) and \( E(\bar{I}_s) = \frac{Q_s}{2}, \) \( \forall s \in N, \) denotes the average value of \( \bar{I}_s \) over the time interval \([t_s, \infty], \) and \( t_s \) being the time that echelon \( s \) receives its first batch \( Q_s. \) When the AP is set at the start of sales for echelon \( k \in N, \) the annuity stream function \( AS^k_i \) is given by \( AS^k_i \).

**Corollary 1.** The SC function can be found as the sum and is independent of the transfer prices \((w_1 \text{ to } w_{n-1})\)

**Corollary 2.** For zero lead-times, the functions are independent of the Anchor Point
Results: zero lead-times

**Theorem 3.** A first order approximation of the average logistics costs $AC_i$ relevant for echelon $i$, $\forall i \in N$, in the classic serial multi-echelon SC with zero lead-times, when adopting Axioms 1 and 2, is:

$$AC_i = s_i \frac{y}{Q_i} + h_i E(I_i) - (h_i(w_{i-1}) - h_i)E(\Bar{I}_{i-1}) = s_i \frac{y}{Q_i} + h_i E(\Bar{I}_i) - h_i(w_{i-1})E(\Bar{I}_{i-1})$$

(7)

where $E(\Bar{I}_i)$ and $E(I_i)$ are defined as in Section 4.1; and $h_i$ and $h_i(w_{i-1})$ as in Section 4.2.

**Consequences:**

- Installation stock versus echelon stock? Irrelevant question
- An echelon must look at its own inventory levels AND the echelon stock of its successor
- Inventory level and echelon stock can be found from tracking own purchases and Point of Sales Data (POS) only
- Unit holding cost: derivable from private information only
- Unit holding reward: derivable from private information only
Results: zero lead-times

**Corollary 3.** A first order approximation of the average logistics costs relevant to the collective of all echelons in a serial multi-echelon SC with zero lead-times, when adopting Axioms 1-3, is:

\[
AC = \sum_{i=1}^{n} AC_i = \sum_{i=1}^{n} \left[ s_i \frac{y}{Q_i} + (h_i - h_i(w_i))E(\tilde{I}_i) \right] + h_n(w_n)E(\tilde{I}_n)
\]  

where \( AC_i \) is given by Eq.(7). It holds that \( AC \) is independent of the transfer prices \( W \setminus \{w_0,w_n\} \).

- SC function = sum of the individual functions
- SC function is independent of transfer prices
- SC function uses unit echelon holding costs and echelon stocks (as in Clark & Scarf)
Results: zero lead-times

Corollary 4. The classic solution for AC given by Eq. (3) or Eq. (4) and based on the classic definitions for $h_i$ and $\tilde{h}_i$ in Section 4.1 where $\tilde{h}_i \equiv \alpha_i e_i + f_i$ for $i \in N \setminus \{n\}$, $\tilde{h}_n \equiv \alpha_n (e_n + w_n) + f_n$, and $\alpha_i = \alpha$, $\forall i \in N$, is equal to the function $AC$ of Corollary 3 in which $h_i$ and $h_i(w_i)$ are defined as in Section 4.2.

• Clark and Scarf’s unit echelon holding costs can be derived from local information:
  Consider the holding cost from all per unit made outgoing cash payments
  Subtract $\alpha$ times the unit price you pay your supplier
  $= $ your unit echelon holding cost
Corollary 5. The necessary condition for the classic installation costs $AC_i$ as defined in Section 4.1 to become equal to the costs $AC_i$ as defined in Theorem 3 is that each echelon $i \in N \setminus \{1\}$ must charge a unit price that is equal to its unit costs, i.e. $w_{i-1} = w_i + e_i + f_i/\alpha_i$. (It is assumed that $\alpha_i \neq 0$.) The sufficient conditions for equality are then for the classic unit holding costs to be $h_i = \sum_{j=i}^{n}(\alpha e_j + f_j) + \alpha w_n$, where $\alpha_i = \alpha$, $\forall i \in N$.

Corollary 6. The necessary condition for the classic echelon costs $\widehat{AC}_i$ as defined in Section 4.1 to become equal to the costs $AC_i$ as defined in Theorem 3 is that each echelon $i \in N \setminus \{1\}$ charges a zero unit price. (It is assumed that $\alpha_i \neq 0$.) The sufficient conditions for equality are then for the classic unit echelon holding costs to be $\widehat{h}_i = \alpha_i e_i + f_i$, $i \in N \setminus \{n\}$, and $\widehat{h}_n = \alpha_n(w_n + e_n) + f_n$.

• The installation stock functions are compatible with the AS functions only when you charge your buyer at your unit costs (means: your firm incurs a loss)
• The echelon stock functions are compatible with the AD functions only when you charge your buyer a zero price (means: your firm incurs a loss)
• The new framework is a proper generalisation of classic multi-echelon theory towards any price vector $W$. 

Results: zero lead-times
Applications

- 1. Joint lot-size optimisation and side-payments (Goyal 1976)

- 2. Price discounts to increase the vendor’s profits (Joglekar 1988)

- 3. Perfect coordination through VMI+ (Bernstein et al. 2006)
Table 1: Problem instance characteristics for the buyer-vendor supply chain

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<th>s_2</th>
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for each instance: \( \alpha = 0.2 \)

Table 2: Solutions for optimisation in the buyer-supplier supply chain

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<td>6</td>
<td>NPV</td>
<td>993</td>
<td>16</td>
<td>10,075</td>
<td>59,473</td>
<td>69,548</td>
<td>2</td>
<td>7,769</td>
<td>40,071</td>
<td>25,576</td>
<td>65,647</td>
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<tr>
<td>6</td>
<td>Classic I</td>
<td>993</td>
<td>16</td>
<td>10,075</td>
<td>62,377</td>
<td>72,452</td>
<td>12</td>
<td>1,293</td>
<td>10,429</td>
<td>61,738</td>
<td>72,166</td>
</tr>
</tbody>
</table>
Application 1

- Incentive scheme to adopt SC optimum
  - Final cost distribution vector must be an imputation
  - Assume: Shapley value
  - Leads to Nash equilibrium

In order for both parties to accept the SC optimum, side-payments must be organised such that the final cost distribution vector is an imputation (a sufficient condition for a two-player cooperative game). If the Shapley value solution is selected, firm $k$ is to receive a side-payment of $SP_k = 0.5 \triangle AC - (AC_k^{**} - AC_k^*) > 0$ from the other firm, where $\triangle AC = AC^{**} - AC^* \leq 0$, $k \in \{1, 2\}$. (If $SP_k < 0$, firm $k$ pays the other firm.) Every firm $k$ will receive the final costs $AC'_k = AC_k^* + 0.5 \triangle AC \leq AC_k^*$. Each firm will now prefer the solution that minimises the costs across the SC, as this is also the one that minimises their individual costs (the solution is thus also a Nash equilibrium).
Application 1

The classic framework is difficult to be reconciled with the AS framework.

Table 3. Optimisation benefits and side-payments in the buyer-supplier supply chain

<table>
<thead>
<tr>
<th>#</th>
<th>Method</th>
<th>( \Delta AC(%) )</th>
<th>( SP_1 )</th>
<th>( SP_2 )</th>
<th>( AC_1' )</th>
<th>( AC_2' )</th>
<th>( \Delta AC(%)(E) )</th>
<th>( AC_1'(E) )</th>
<th>( AC_2'(E) )</th>
<th>( I?(E) )</th>
<th>( F?(E) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>NPV</td>
<td>-11.45</td>
<td>-317</td>
<td>317</td>
<td>217</td>
<td>579</td>
<td>-3.27</td>
<td>217</td>
<td>679</td>
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<td>No</td>
</tr>
<tr>
<td>2</td>
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<td>1.74</td>
<td>-72</td>
<td>72</td>
<td>246</td>
<td>635</td>
<td>7.35</td>
<td>246</td>
<td>586</td>
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<tr>
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<td>-901</td>
<td>901</td>
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<td>467</td>
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<td>329</td>
<td>1,263</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>2</td>
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<td>-0.77</td>
<td>-21</td>
<td>21</td>
<td>411</td>
<td>688</td>
<td>-5.48</td>
<td>411</td>
<td>505</td>
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<td>No</td>
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<tr>
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<td>-316</td>
<td>316</td>
<td>2,283</td>
<td>-757</td>
<td>9.89</td>
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<td>323</td>
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<tr>
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<td>Classic I</td>
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<td>0</td>
<td>0</td>
<td>2,360</td>
<td>12</td>
<td>-0.20</td>
<td>2,360</td>
<td>-683</td>
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<td>No</td>
</tr>
<tr>
<td>4</td>
<td>NPV</td>
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<td>-2,281</td>
<td>2,281</td>
<td>1,367</td>
<td>1,262</td>
<td>31.92</td>
<td>1,367</td>
<td>3,408</td>
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<td>No</td>
</tr>
<tr>
<td>4</td>
<td>Classic I</td>
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<td>-22</td>
<td>22</td>
<td>1,601</td>
<td>1,996</td>
<td>-2.67</td>
<td>1,601</td>
<td>1,436</td>
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<tr>
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<td>-805</td>
<td>805</td>
<td>1,141</td>
<td>1,230</td>
<td>2.23</td>
<td>1,141</td>
<td>1,753</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>5</td>
<td>Classic I</td>
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<td>-108</td>
<td>108</td>
<td>1,233</td>
<td>1,518</td>
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<td>1,233</td>
<td>1,248</td>
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<td>No</td>
</tr>
<tr>
<td>6</td>
<td>NPV</td>
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<td>-31,947</td>
<td>31,947</td>
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<td>57,522</td>
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<td>8,124</td>
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<td>No</td>
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<td>6</td>
<td>Classic I</td>
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<td>-497</td>
<td>497</td>
<td>9,932</td>
<td>62,235</td>
<td>-1.67</td>
<td>9,932</td>
<td>58,453</td>
<td>Yes</td>
<td>No</td>
</tr>
</tbody>
</table>

\( (E) \) When evaluating solution and side-payments through the eyes of the other framework;

\[ I = \text{Imputation}; \quad F = \text{Fair (Shapley value)} \]
### Application 2

#### Table 4  
**Optimal price-discount schemes for the vendor**

<table>
<thead>
<tr>
<th>#</th>
<th>Method</th>
<th>( \gamma^* )</th>
<th>( \beta^* )</th>
<th>( Q_{1}^{**} )</th>
<th>( m_2(Q_{1}^{**}) )</th>
<th>( \Delta \text{Profits(%)} )</th>
<th>( \Delta \text{Profits(%)(E)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>NPV</td>
<td>3.65</td>
<td>0.986</td>
<td>3,267</td>
<td>1</td>
<td>3.45</td>
<td>1.29</td>
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<tr>
<td>1</td>
<td>Classic I</td>
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<td>0.997</td>
<td>1,674</td>
<td>2</td>
<td>1.50</td>
<td>2.35</td>
</tr>
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<td>0.981</td>
<td>3,399</td>
<td>1</td>
<td>0.61</td>
<td>-1.72</td>
</tr>
<tr>
<td>2</td>
<td>Classic I</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>&lt;&lt;0.1</td>
<td>/</td>
</tr>
<tr>
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<td>NPV</td>
<td>1.53</td>
<td>0.994</td>
<td>6,226</td>
<td>1</td>
<td>0.72</td>
<td>-1.00</td>
</tr>
<tr>
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<td>Classic I</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>&lt;&lt;0.1</td>
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<tr>
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<td>NPV</td>
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<td>0.968</td>
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<td>1</td>
<td>1.24</td>
<td>-2.77</td>
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<tr>
<td>4</td>
<td>Classic I</td>
<td>/</td>
<td>/</td>
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<td>&lt;&lt;0.1</td>
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<td>1.93</td>
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<td>0.998</td>
<td>219</td>
<td>2</td>
<td>0.60</td>
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<td>NPV</td>
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<td>0.994</td>
<td>7,815</td>
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<td>0.13</td>
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<tr>
<td>6</td>
<td>Classic I</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>&lt;&lt;0.1</td>
<td>/</td>
</tr>
</tbody>
</table>

(E) When evaluating price discount scheme through the eyes of the other framework.

- Price discounts schemes are more profitable
- Lot-size increases more than that price decreases: supplier’s reward increases
Consignment `solves' the transfer price mismatch, but leaves the supplier with no supplier rewards

The SC optimum in VMI+ is now compatible with the approach based on AS functions

The misallocation of SC-wide savings using the classic framework is still considerable
Conclusions

- The Anchor Point approach shows that in some systems, multiple NPV functions are possible, leading to different holding cost terms, and inventory decisions.

- Arguably, the best match between NPV and classic inventory models occurs under PULL conditions (AP at the output of the system).

- Even under PULL conditions, the classic functions may not be equivalent to linear approximations of NPV functions. Typically, supplier rewards are not considered in classic functions.

- NPV gives answers to the role of holding costs in profit functions.
Conclusions

- Whenever a stocking point has buyers who make lot-size decisions, the stocking point must consider two types of unit holding costs
  - The ‘classic’ unit holding cost, based on investments
  - A unit holding ‘reward’, based on unit profits (revenues)
- The profit function of the stocking point is (in general) determined by its relative position to the Anchor Point
- Even with a properly placed AP and a properly ‘adjusted’ unit holding cost, the classic approach of arriving at holding costs by integration of inventory levels over a cycle does lead in general to equivalence with a linear NPV function
Conclusions

- The NPV framework recognises good features in each of the three classic approaches:
  - Classic I: including the real price paid to suppliers
  - Classic II: the multi-echelon theory of Clark and Scarf identifies the SC function
  - Classic III: Crowther’s supplier function is correct and applies in general (when properly adapted for more than 2 echelons)
- The NPV framework gives an approach that unifies the classic approaches, and allows extensions (non-zero lead-times, negative transfer prices, different anchor points, ...
References


P.Beullens@soton.ac.uk
\[ H = \frac{h}{T} \int_0^T I(t) \, dt = hE(I) \]